
LESS

LINEAR EQUIVALENCE SIGNATURE SCHEME



SAPIENZA
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+CrypTO

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LESSon 0:

**LESS:
THE ORIGINS**



2017 - CALL FOR PROPOSALS

*“NIST is soliciting proposals for post-quantum cryptosystems [...].
The goal of this process is to select a number of acceptable candidate
cryptosystems for standardization.”*

2024: standardisation of



CRYSTALS-Dilithium



CRYSTALS-KYBER



SPHINCS+

2020 - LESS!

**LESS is More: Code-Based Signatures
without Syndromes**

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*«[...] we construct a signature scheme by exploring a new approach to the area.
[...] We show that practical instances of our protocol have the potential to
outperform the state of the art on code-based signatures [...].»*

2022 - CALL FOR SIGNATURES

«NIST is calling for additional digital signature proposals to be considered in the PQC standardization process.»

Requirements:

1. not based on structured lattices
2. performance advantage over **SPHINCS+**
3. if lattice-based, performance advantage over **CRYSTALS**





WHAT IS **LINEAR EQUIVALENCE**?



WHAT IS **LESS**?



IS **LESS** SECURE?

*...WHAT IS A **CODE**?*



WHAT IS **LINEAR EQUIVALENCE**?



WHAT IS **LESS**?



IS **LESS** SECURE?

*...WHAT IS A **CODE**?*

1 WHAT IS **LINEAR EQUIVALENCE**?

2 WHAT IS **LESS**?

3 IS **LESS** SECURE?

LESSon 1:

**LINEAR CODE
EQUIVALENCE**

LINEAR CODES

An (n, k) - **linear code** C is a k -dimensional subspace of F_q^n .

The matrix G whose rows are a basis of C is its **generator matrix**.

LINEAR CODE

All generator matrices are connected by some change of basis $S \in GL_k(q)$.

For some S , $SG = (I_k \mid A)$. This is the **systematic form**.

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The dual of an (n, k) -linear code C is the $(n, n - k)$ -linear code

$$C^\perp = \{y \in F_q^n : \forall x \in C, yx^T = 0\}$$

DUAL CODE

Its generator matrix is the **parity check matrix** of C .

If $G = (I_k \mid A)$, then $H = (-A^T \mid I_{n-k})$.

LINEAR CODES

$$GH^T = \begin{array}{|c|c|} \hline I_k & A \\ \hline \end{array} \begin{array}{|c|} \hline -A \\ \hline I_{n-k} \\ \hline \end{array} = 0$$

Its generator matrix is the **parity check matrix** of C .

If $G = (I_k \mid A)$, then $H = (-A^T \mid I_{n-k})$.

MEET: MONOMIAL MATRICES

Permutation

$$\pi \in S_n$$

$$\begin{array}{|c|} \hline 1 & & \\ \hline & 1 & \\ \hline 1 & & \\ \hline \end{array} \begin{array}{|c|} \hline x_1 \\ \hline x_2 \\ \hline x_3 \\ \hline \end{array} = \begin{array}{|c|} \hline x_3 & x_2 & x_1 \\ \hline \end{array}$$

Linear isometry

$$\mu = (v; \pi) \in \underbrace{F_q^{*n} \rtimes S_n}_{M_n}$$

$$\begin{array}{|c|} \hline v_1 & & \\ \hline & v_2 & \\ \hline v_3 & & \\ \hline \end{array} \begin{array}{|c|} \hline x_1 \\ \hline x_2 \\ \hline x_3 \\ \hline \end{array} = \begin{array}{|c|} \hline v_3 x_3 & v_2 x_2 & v_1 x_1 \\ \hline \end{array}$$

MEET: MONOMIAL MATRICES

Permutation Equivalence Problem (search)

given C, C'

find $\pi \in S_n$ such that $C' = \pi(C)$

Linear Equivalence Problem (search)

given C, C'

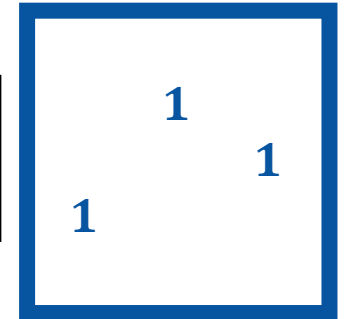
find $\mu \in M_n$ such that $C' = \mu(C)$

MEET: MONOMIAL MATRICES

Permutation Equivalence Problem (search)

given G, G'

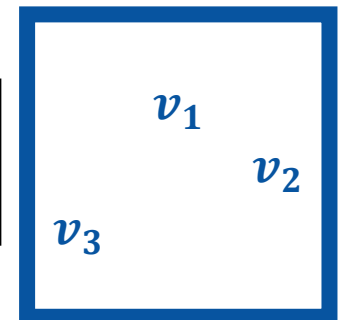
find $P \in S_n$ such that $G' = GP$



Linear Equivalence Problem (search)

given G, G'

find $Q \in M_n$ such that $G' = GQ$

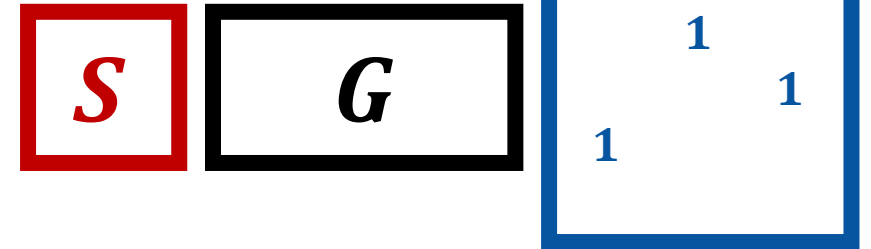


MEET: MONOMIAL MATRICES

Permutation Equivalence Problem (search)

given G, G'

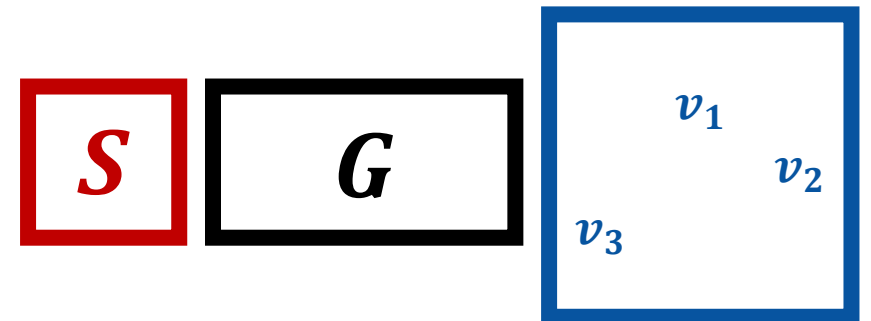
find $P \in S_n$ and $S \in GL_k(q)$ such that $G' = SGP$



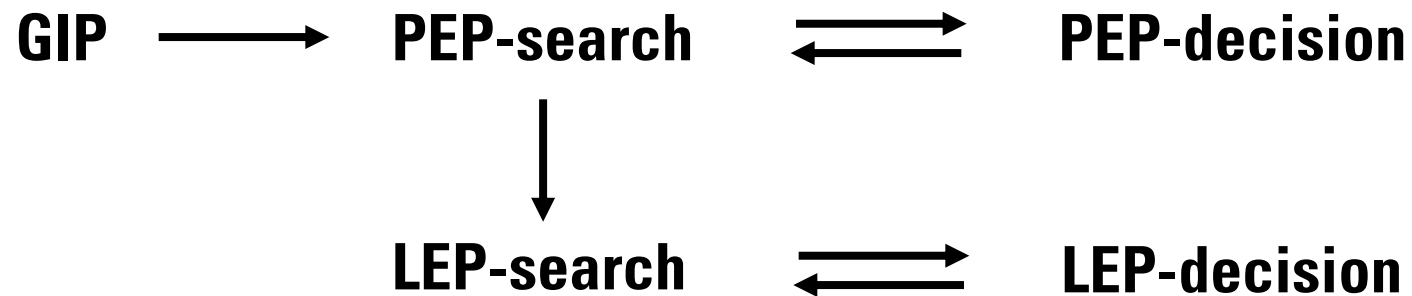
Linear Equivalence Problem (search)

given G, G'

find $Q \in M_n$ and $S \in GL_k(q)$ such that $G' = SGQ$



COMPLEXITY OF LEP & PEP



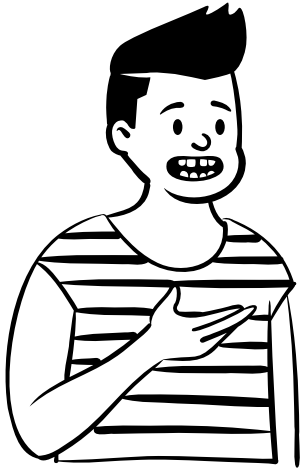
A \longrightarrow **B** A reduces to B

LESSon 2:

**DESIGNING
THE SCHEME**

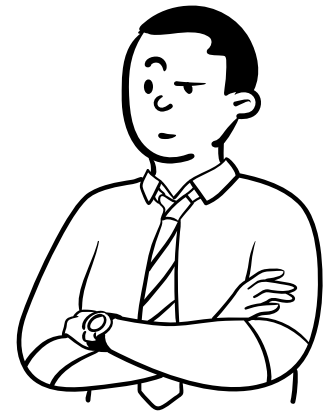


LESS: THE SIGMA PROTOCOL (ZKP_oK)



I know Q and S such that $G' = SGQ!$

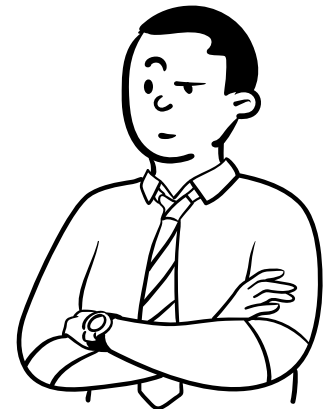
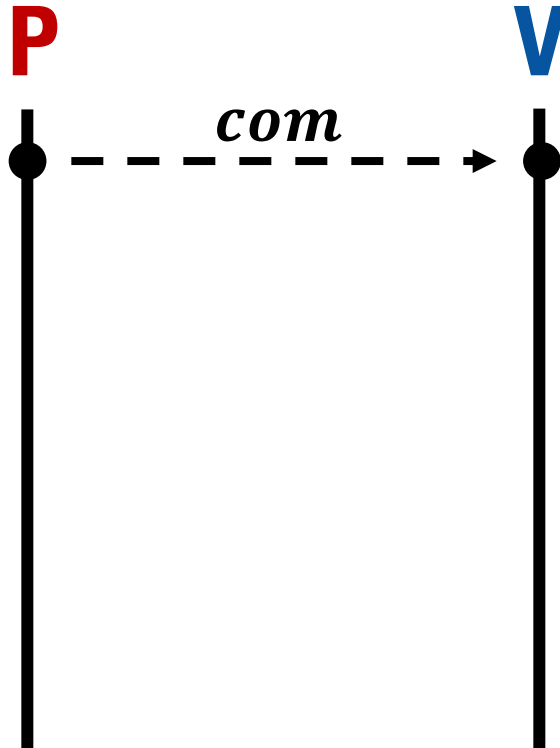
Prove it.



LESS: THE SIGMA PROTOCOL (ZKP_oK)



random $\bar{Q} \in M_n$
 $com = SF(G\bar{Q})$



$Q, S : G' = SGQ$

LESS: THE SIGMA PROTOCOL (ZKP_oK)



random $\bar{Q} \in M_n$
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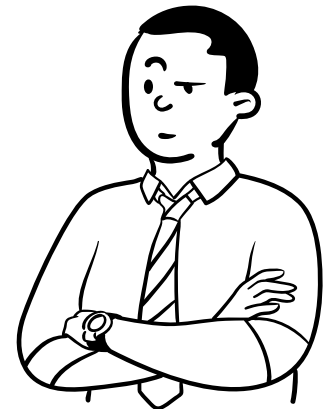
P

com

V

ch

random $ch \in \{0, 1\}$



$Q, S : G' = SGQ$

LESS: THE SIGMA PROTOCOL (ZKP_oK)



random $\bar{Q} \in M_n$
 $com = SF(G\bar{Q})$

if $ch = 0$, $rsp = \bar{Q}$
if $ch = 1$, $rsp = Q^{-1}\bar{Q}$

$Q, S : G' = SGQ$

P

V

com

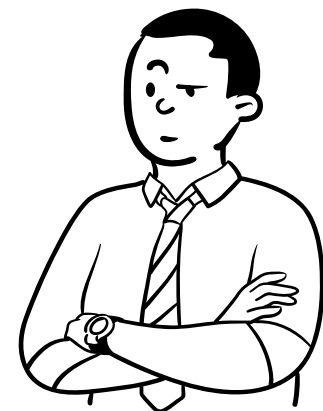
ch

rsp

random $ch \in \{0, 1\}$

$SF(G \cdot rsp) = com$

$SF(G' \cdot rsp) = com$



LESS: THE SIGMA PROTOCOL (ZKP_{oK})



random $\bar{Q} \in M_n$
 $com = h(SF(G\bar{Q}))$

if $ch = 0$, $rsp = \bar{Q}$
if $ch = 1$, $rsp = \bar{Q}^{-1}$

$Q, S : G' = SGQ$

P

com

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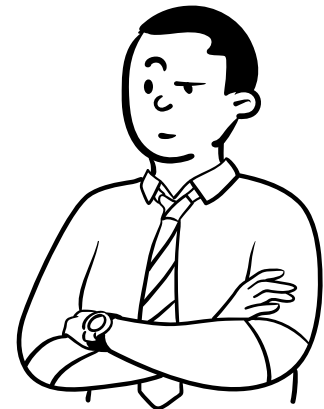
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$h(SF(G' \cdot rsp)) = com$



LESS: THE SIGMA PROTOCOL (ZKP_{oK})



random $\bar{Q} \in M_n$
 $com = h(RREF(G\bar{Q}))$

if $ch = 0$, $rsp = \bar{Q}$
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com

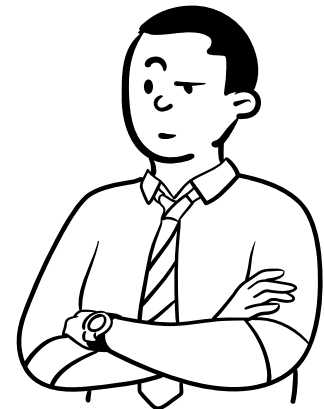
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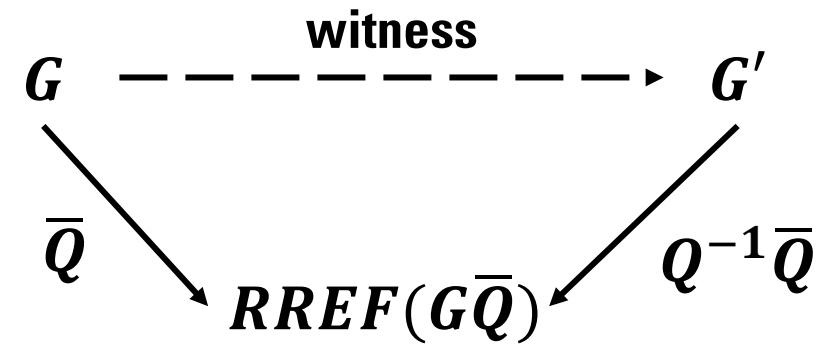
random $ch \in \{0, 1\}$

$h(RREF(G \cdot rsp)) = com$
 $h(RREF(G' \cdot rsp)) = com$



Q: $G' = RREF(SQ)$

LESS: THE SIGMA PROTOCOL (ZKP_oK)



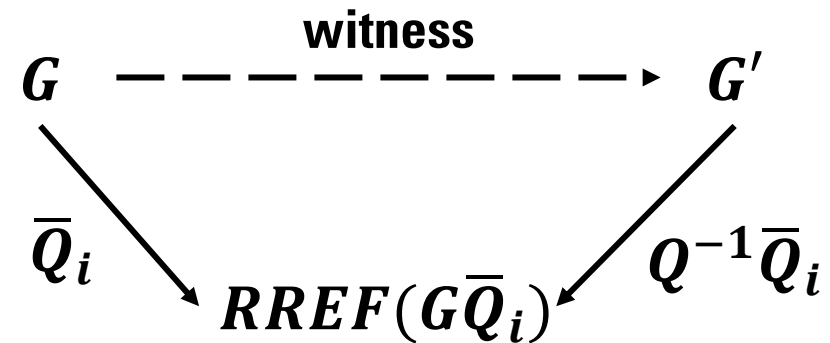
LESS: THE SIGNATURE

random $\bar{Q} \in M_n$
 $com_i = h(RREF(G\bar{Q})) \quad i = 1, \dots, t$

$ch = h(m, com)$
 $ch_i = ch[i] \quad i = 1, \dots, t$

for $i = 1, \dots, t$
 if $ch_i = 0, rsp_i = \bar{Q}$
 if $ch_i = 1, rsp_i = Q^{-1}\bar{Q}$

$\sigma \leftarrow (com_1, \dots, com_t, rsp_1, \dots, rsp_t)$



soundness of Σ is $\frac{1}{2}$,
iterate $t = \lambda$ times

LESSon 3:

**SECURITY &
ATTACKS**



SECURITY PROOF

If Σ is a non-trivial canonical identification protocol secure against passive impersonation attacks, the signature scheme $FS(\Sigma)$ is UF-CMA secure,

$$\text{“ } Adv_{FS(\Sigma),A}^{uf-cma}(\lambda) \leq f(\lambda) \cdot Adv_{\Sigma,B}^{pa-imp}(\lambda) + g(\lambda) \text{”}$$

FACT

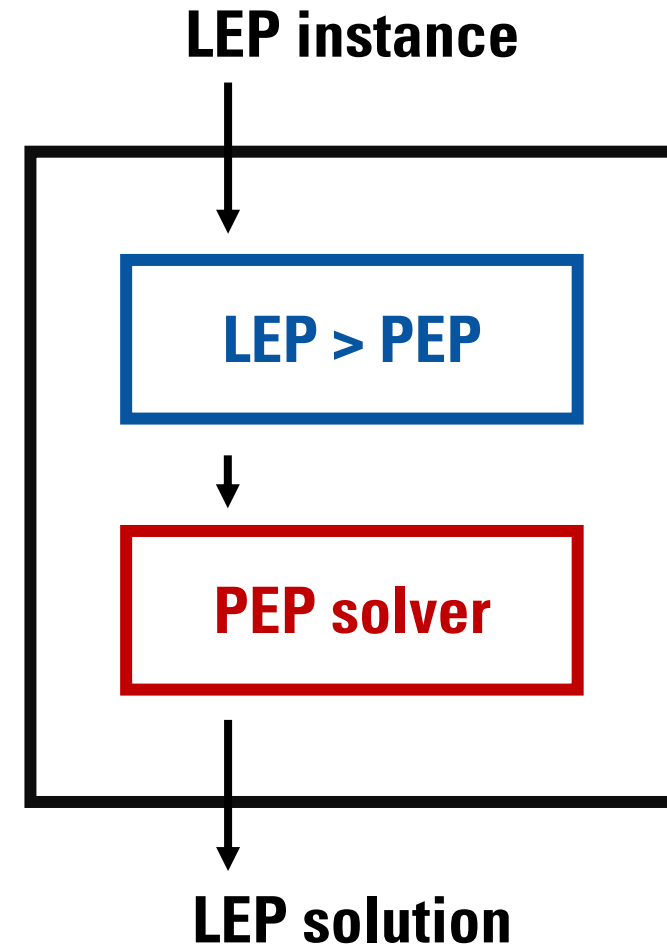
This holds in **ROM** and is believed to hold for the **QR**OM.

The security of LESS is based on that of LEP.

uf-cma sec. of $FS(\Sigma)$ \longrightarrow **pa-imp sec. of Σ** \longrightarrow **LEP-search**

ATTACKS

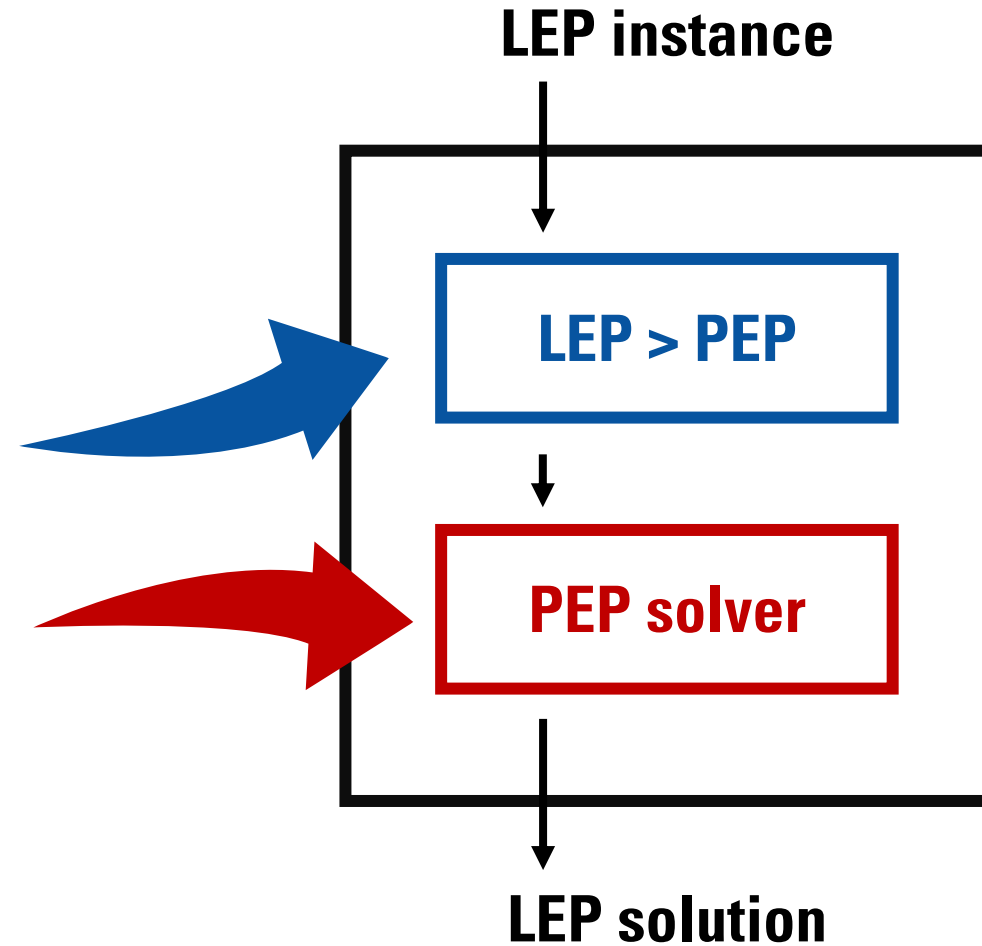
Type 1: solving PEP (e.g. SSA)



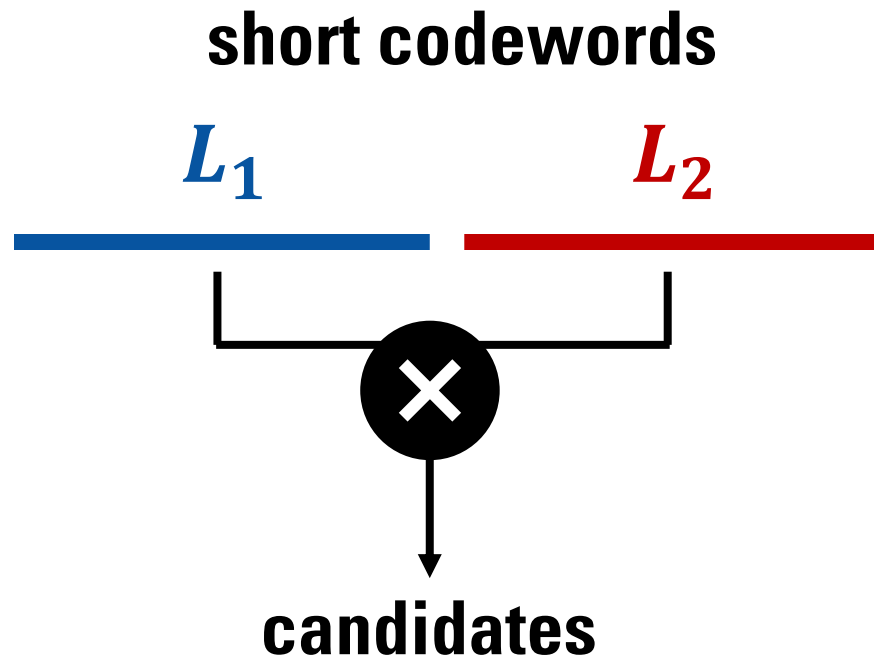
ATTACKS

Type 1: solving PEP (e.g. SSA)

- if two codes are linearly equivalent, their **closures** are permutationally equivalent
 $cl(C)_a = \{c \otimes a : c \in C\}$ (a ordering of F_q^*)
- deteriorates with dimension of the **hull** of the code, but closures have maximal hull...



ATTACKS



Type 2: low-weight codeword finding (e.g. Prange)

- in general of exponential complexity
- structured variants deteriorate with increasing q
- Leon's algorithm: generate relations with L_1, L_2 of weight- w codewords such that $L_1 = QL_2$
- NIST constraints the depth of quantum circuits, rendering quantum attacks (e.g. Prange + Grover) impractical

ATTACKS

| Type of equivalence | Algorithm | Complexity | Notes |
|---------------------|-----------|--------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------|
| Permutation | Leon | $O(C_{ISD}(q, n, k, d_{GV}) \cdot 2 \ln(N_w))$ | Preferable with small finite fields and large hulls. |
| | Beullens | $O\left(\frac{2L \cdot C_{ISD}(q, n, k, w)}{N_w(1 - 2^{L \log_2(1 - L/N_w)})}\right)$ | Preferable with large finite fields and large hulls. It may fail, when L is too small. |
| | SSA | $O(n^3 + n^2 q^h \log n)$ | Efficient with small, non-trivial hulls |
| | BOS | $\begin{cases} O(n^{2.373} C_{WGI}(n)) & \text{if } h = 0 \\ O(n^{2.373+h+1} C_{WGI}(n)) & \text{if } h > 0 \end{cases}$ | Efficient with trivial hulls |
| Linear | Leon | $O(C_{ISD}(q, n, k, d_{GV}) \cdot 2 \ln(N_w))$ | Preferable with small finite fields and large hulls. |
| | Beullens | $O\left(\frac{2L \cdot C_{ISD}(q, n, k, w)}{N_w(1 - 2^{L \log_2(1 - L/N_w)})}\right)$ | Preferable with large finite fields and large hulls. It may fail, when L is too small. |
| | SSA | $\begin{cases} O(n^3 + n^2 q^h \log n) & \text{if } q < 5 \\ O(n^3 + n^2 q^k \log n) & \text{if } q \geq 5 \end{cases}$ | Efficient if $q < 5$ and the hull is trivial. |

Table 2: Summary of techniques to solve the code equivalence problem

PARAMETERS

$$\begin{aligned} Adv_{FS(\Sigma),A}^{uf-cma}(\lambda) &\leq f(\lambda) \cdot Adv_{\Sigma,B}^{pa-imp}(\lambda) + g(\lambda) \\ &\leq f'(\lambda) \cdot Adv_C^{LEP}(\lambda) + g'(\lambda) \end{aligned}$$

PARAMETERS

$$\begin{aligned} Adv_{FS(\Sigma),A}^{uf-cma}(\lambda) &\leq f(\lambda) \cdot Adv_{\Sigma,B}^{pa-imp}(\lambda) + g(\lambda) \\ &\leq f'(\lambda) \cdot Adv_C^{LEP}(\lambda) + g'(\lambda) \end{aligned}$$

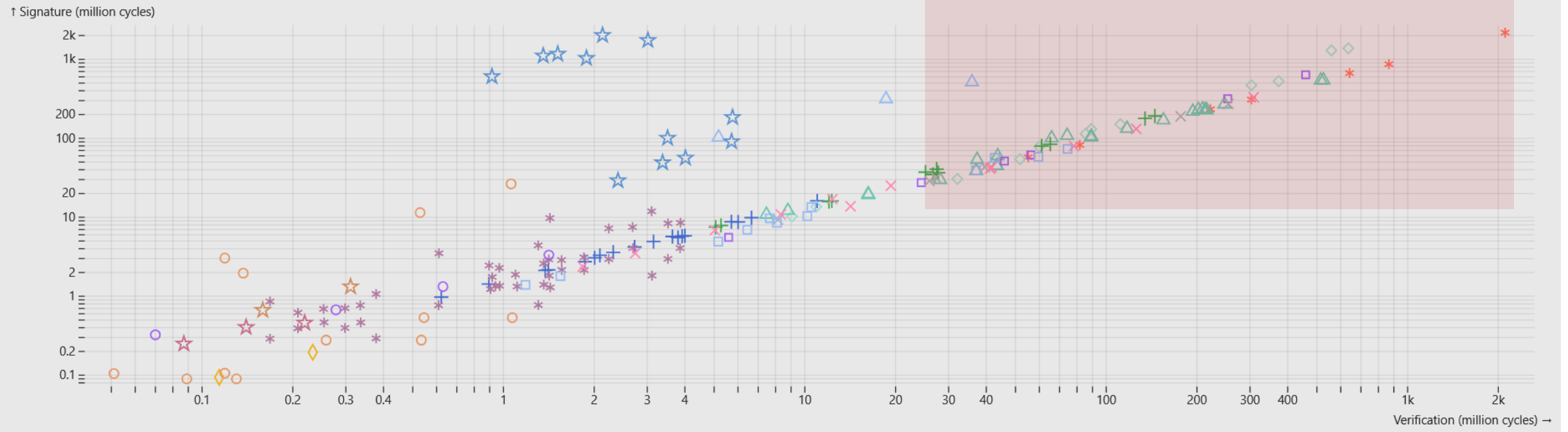
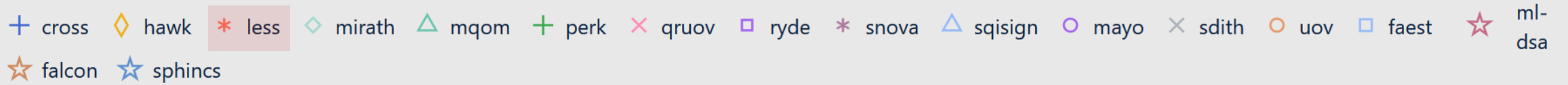
INSTANTIATION

Consider $q \geq 5$ and random codes. We select n, k, q such that for any weight $w = 1, \dots, n$ finding lists of weight- w codewords L_1 and L_2 having non-empty $L_1 \cap L_2$ takes at least time 2^λ .

$$\frac{C_{ISD}(w)}{\sqrt{N(w)}} < 2^\lambda$$

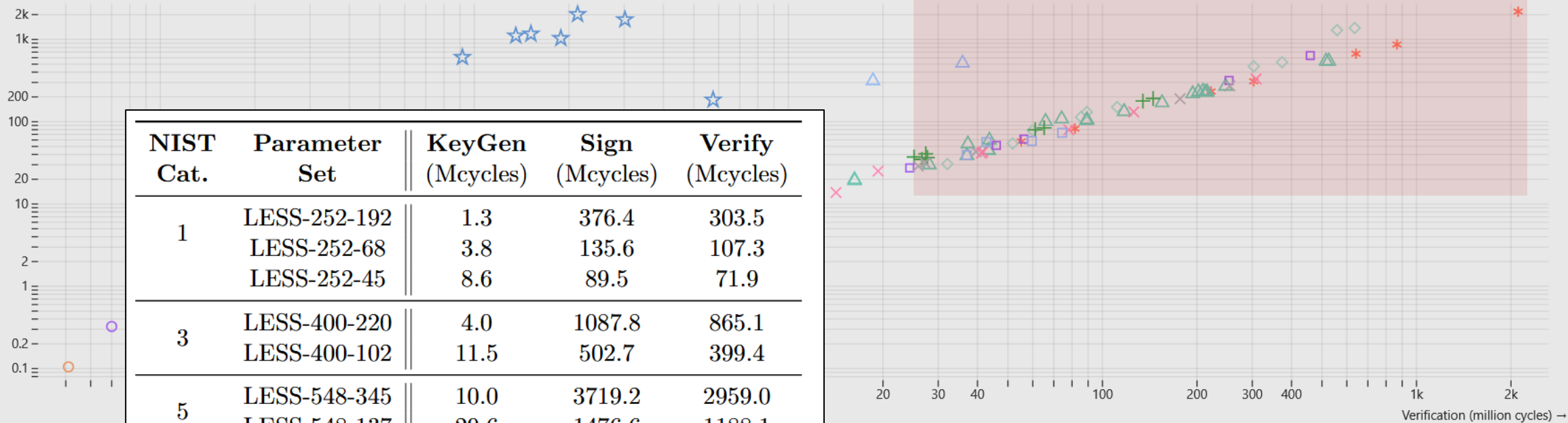


FOR AN OLD VERSION OF LESS!



+ cross ◇ hawk * less ◇ mirath △ mqom + perk × quov □ ryde * snova △ sqisign ○ mayo × sdith ○ uov □ faest ☆ ml-dsa
 ☆ falcon ☆ sphincs

↑ Signature (million cycles)



| NIST Cat. | Parameter Set | KeyGen (Mcycles) | Sign (Mcycles) | Verify (Mcycles) |
|-----------|---------------|------------------|----------------|------------------|
| 1 | LESS-252-192 | 1.3 | 376.4 | 303.5 |
| | LESS-252-68 | 3.8 | 135.6 | 107.3 |
| | LESS-252-45 | 8.6 | 89.5 | 71.9 |
| 3 | LESS-400-220 | 4.0 | 1087.8 | 865.1 |
| | LESS-400-102 | 11.5 | 502.7 | 399.4 |
| 5 | LESS-548-345 | 10.0 | 3719.2 | 2959.0 |
| | LESS-548-137 | 29.6 | 1476.6 | 1188.1 |



NEW VERSION OF LESS!

*...WHAT IS A **CODE**?*

1 WHAT IS **LINEAR EQUIVALENCE**?

2 WHAT IS **LESS**?

3 IS **LESS** SECURE?

LESSon 4:

UPGRADES!



LESSon 4:

UPGRADES!

... an overview

LESS-F

random $\bar{Q} \in M_n$

$com_i = h(RREF(G\bar{Q})) \quad i = 1, \dots, t$

$ch = h(m, com)$

$ch_i = ch[i] \quad i = 1, \dots, t$

for $i = 1, \dots, t$

if $ch_i = 0, rsp_i = \bar{Q}$

if $ch_i = 1, rsp_i = Q^{-1}\bar{Q}$

$\sigma \leftarrow (com_1, \dots, com_t, rsp_1, \dots, rsp_t)$

idea: challenge is lighter for $b = 0$, just send the seed used to generate \bar{Q}

Use a weight-restricted hash h .



need more rounds t



more efficient broadcasts

LESS-M

random $\bar{Q} \in M_n$
 $com_i = h(RREF(G\bar{Q})) \quad i = 1, \dots, t$


$ch = h(m, com)$
 $ch_i = ch[(i-1)\ell, i\ell] \quad i = 1, \dots, t$

for $i = 1, \dots, t$
if $ch_i = 1, rsp_i = Q_{ch_i}^{-1} \bar{Q}$

$\sigma \leftarrow (com_1, \dots, com_t, rsp_1, \dots, rsp_t)$

idea: increasing the challenge space
reduces repetitions t

The ch_i become ℓ - bit challenges,
interpreted as integers in $[0, 2^\ell - 1]$.

 2^ℓ public keys Q_i (note: $Q_1 = I_n$)

 reduced rounds t

LESS-FM

LESS-F + LESS-M

| Optimization Criterion | LESS Type | n | k | q | ℓ | t | ω | pk (kB) | sig (kB) | pk + sig (kB) |
|------------------------|-----------|-----|-----|-----|--------|-----|----------|---------|----------|---------------|
| Min. pk size | F Mono | 198 | 94 | 251 | 1 | 283 | 28 | 9.77 | 15.2 | 24.97 |
| Min. sig size | FM Perm | 235 | 108 | 251 | 4 | 66 | 19 | 205.74 | 5.25 | 210.99 |
| Min. pk + sig size | F Perm | 230 | 115 | 127 | 1 | 233 | 31 | 11.57 | 10.39 | 21.96 |
| Beullens [14] | - Mono | 250 | 125 | 53 | 1 | 128 | - | 11 | 28 | 39 |

Table 7: Parameter sets for LESS-FM, for a security level of $\lambda = 128$ classical bits.

IS-LESS

idea: for $b = 1$, just consider the action of $Q^{-1}\bar{Q}$ on an information set J

But the verifier needs to compute the same code.

coordinates in J : equal up to invertible matrix

$$\bar{G}_J' = S\bar{G}_J$$

coordinates outside J : equal up to invertible and monomial matrices

$$\bar{G}_{[n]\setminus J}' = S\bar{G}_{[n]\setminus J}Z$$

solution: compute a «canonical form»

prover: $RREF(\bar{G}_J)$ w.r.t. J \longrightarrow $V = \bar{G}_J^{-1}\bar{G}_{[n]\setminus J}$ \longrightarrow scale & sort columns in lexicographical order

$\#$
verifier: $RREF(\bar{G}_J')$ w.r.t. J \longrightarrow $V' = \bar{G}_J^{-1}\bar{G}_{[n]\setminus J}Z$ \longrightarrow scale & sort columns in lexicographical order

INTERLUDE: CRYPTOGRAPHIC GROUP ACTIONS

$$\begin{aligned} \star & : G \times X \rightarrow X \\ (g, x) & \mapsto x \star g \end{aligned}$$

Cryptographic if:

- **effective** (efficient sampling, membership testing, evaluation)
- pseudorandom outputs
- one-way
- ...

INTERLUDE: CRYPTOGRAPHIC GROUP ACTIONS

$$\begin{aligned} \star &: G \times X \rightarrow X \\ ((S; (\alpha, Q)), A) &\mapsto S\alpha(GQ) \end{aligned}$$

$$G = GL_k(q) \rtimes (\text{Aut}(F_q) \times M_n)$$

$X \subset F_q^{k \times n}$ full-rank matrices

INTERLUDE: CRYPTOGRAPHIC GROUP ACTIONS

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$G = GL_k(q) \rtimes (Aut(F_q) \times M_n)$ monomial operations & change of basis
 $X \subset F_q^{k \times n}$ full-rank matrices code generators

INTERLUDE: CRYPTOGRAPHIC GROUP ACTIONS

$$\begin{aligned} \star &: G \times X \rightarrow X \\ ((S; (\alpha, Q)), A) &\mapsto S\alpha(GQ) \end{aligned}$$

$$G = GL_k(q) \rtimes (\text{Aut}(F_q) \times M_n)$$

$X \subset F_q^{k \times n}$ full-rank matrices

monomial operations & change of basis

code generators

excellent framework!

CF-LESS

idea: proving that C and C lie in the same equivalence class reduces witness size

$F \leq M_n$ subgroup such that any $\varphi \in F$ is decomposed as $(\varphi_k, \varphi_{n-k}) \in M_k \times M_{n-k}$
any isometry ψ is the permutation of a $\varphi \in F$

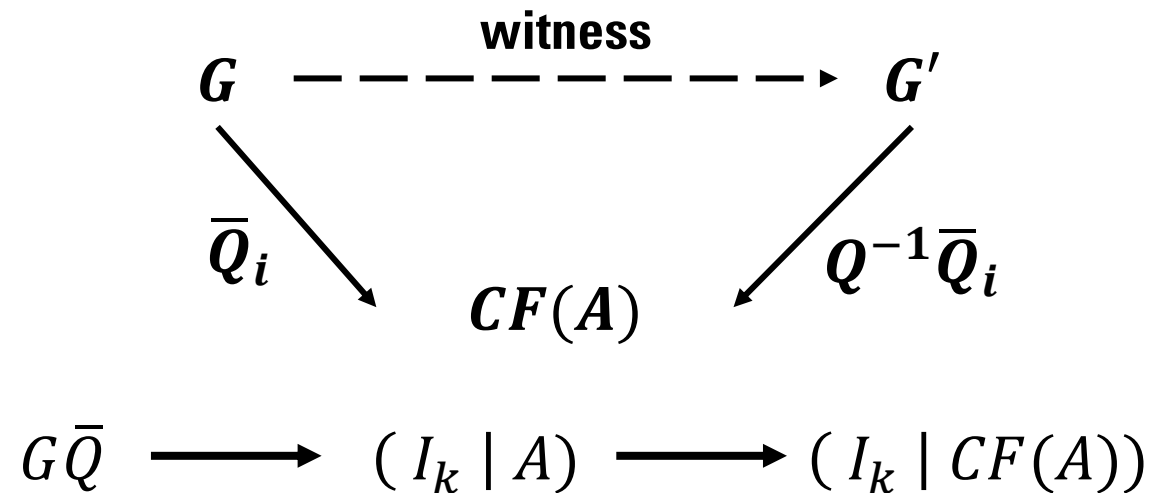
$CF: F_q^{k \times (n-k)} \rightarrow F_q^{k \times (n-k)} \cup \{\perp\}$ canonical form **invariant over F**
 $CF(A) = CF(Q_{\varphi_k} \cdot A \cdot Q_{\varphi_{n-k}})$ for any $\varphi \in F$

$$G\bar{Q} \longrightarrow (I_k \mid A) \longrightarrow (I_k \mid CF(A))$$

only commit $h(CF(A))$, but we need to save the map $\pi: G\bar{Q} \rightarrow RREF(G\bar{Q})$ for the response

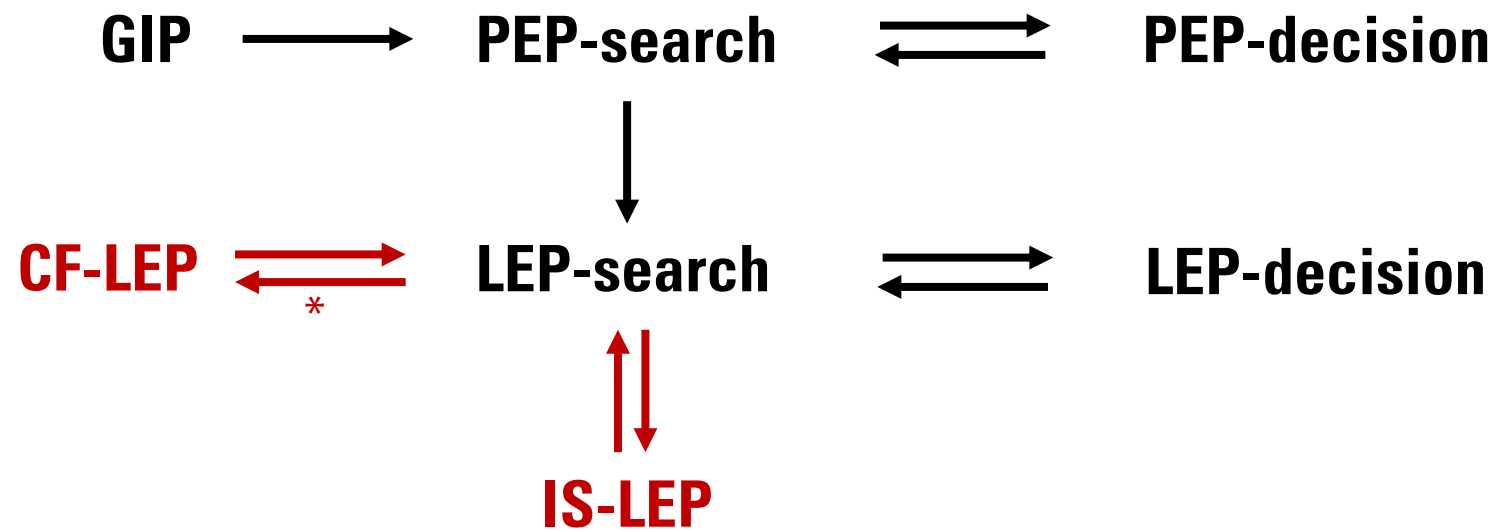
CF-LESS

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only commit $h(CF(A))$, but we need to save the map $\pi: G\bar{Q} \rightarrow RREF(G\bar{Q})$ for the response

CF-LESS



A → **B** A reduces to B

* if *CF* does not fail, i.e. $CF(*) \neq \perp$

TAKE AWAYS

- first code-based signature not using a SDP variation

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- first code-based signature not using a SDP variation
- can adopt the framework of (non-commutative) group actions
 - identity-based signatures
 - ring signatures

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- first code-based signature not using a SDP variation
- can adopt the framework of (non-commutative) group actions
 - identity-based signatures
 - ring signatures
 - *threshold signatures??*



THANKS!

